ELSEVIER Journal of Pure and Applied Algebra 133 (1998) 261–269

JOURNAL OF PURE AND APPLIED ALGEBRA

Critical modules over the second Weyl algebra

S.C. Coutinho

Instituto de Matemática, Universidade Federal do Rio de Janeiro, P.O. Box 68530, 21945-970 Rio de Janeiro, RJ, Brazil

Communicated by C.A. Weibel; received 28 June 1996; received in revised form 19 March 1997

Abstract

We give examples of irreducible modules over the second complex Weyl algebra A_2 whose dual is GK-critical but not irreducible. These modules are then used in the construction of critical modules of length 2 over A_2.

AMS Classification: Primary 16S32; secondary 16E30; 16D60

1. Introduction

In the representation theory of algebras, the rôle of *atom* is played by the irreducible (or simple) modules. This notion has a neat generalization in the concept of *critical* module. Suppose that R is a complex algebra. A finitely generated left R-module M is GK-*critical* if all proper quotients of M have smaller Gelfand–Kirillov dimension than M. The relation with irreducible modules becomes apparent if one uses quotient categories; see [6] for more details.

In 1985 Tauvel asked whether a GK-critical module of finite length over a solvable Lie algebra is necessarily irreducible. This question was answered in the negative by Perets in [9]. Perets used Stafford's example of an irreducible non-holonomic module to construct a GK-critical module of length 2 over the Weyl algebra A_n, for $n \geq 2$. That this example has the required properties is checked by direct computation in the style of Stafford's original example in [10].

In the introduction to his paper, Perets says that, *in principle*, one could show that these modules exist using Ext groups. This may seem unlikely at first, because there is no good duality theory for non-holonomic modules. In this paper we show that the

* E-mail: collier@impa.br.

0022-4049/98/\$ – see front matter
PII: S0022-4049(97)00116-3

existence of GK-criticals of length 2 is, in fact, a consequence of the subtle ways in which the dual of a non-holonomic module misbehaves. The main point is that there exist non-holonomic irreducible modules whose dual is well defined but is *not* irreducible. These will be constructed in Section 2.

2. Irreducible modules

Throughout the paper A will stand for the second complex Weyl algebra. This is the ring of differential operators of two-dimensional affine space. It is generated over $\boldsymbol{C}$ by the coordinate functions x_1, x_2 and by their partial differential operators, denoted by ∂_1, ∂_2.

Let M be a finitely generated left A-module. Suppose that $\mathrm{Ext}^j(M,A)=0$ whenever $j \neq k$, then

$$\mathrm{Ext}^k(\mathrm{Ext}^k(M,A)) \cong M.$$

This suggests that the dual of M should be $\mathrm{Ext}^k(M,A)$. But this is a right A-module. Since it is more convenient to work always with left A-modules, we will use an anti-automorphism to turn the right action of A into a left action. The *standard transposition*, denoted by τ, is the anti-automorphism defined in multi-index notation by $\tau(x^\alpha \partial^\beta) = (-1)^{|\beta|}\partial^\beta x^\alpha$, where $\alpha, \beta \in \boldsymbol{N}^2$. If N is a right module, its transposed is the left module N^τ where $au = u\tau(a)$, for $a \in A$ and $u \in M$. For more details see [5, Section 2, Ch. 16]. Thus, if $\mathrm{Ext}^j(M,A)=0$ for $j \neq k$, the *dual* of M is $M^\star = \mathrm{Ext}^k(M,A)^\tau$.

Two special cases are well known. If $k=0$ the condition on Ext is equivalent to saying that M is projective. In this case, the dual defined above is the usual one, with the action transposed to the left. On the other hand, it follows by Bernstein's inequality [5, Section 4, Ch. 9] that a finitely generated A-module cannot have Gelfand–Kirillov dimension less than 2. Since A satisfies the Auslander condition [3, Section 2, Ch. V], the largest value of k for which the condition on Ext makes sense is 2. Indeed $\mathrm{Ext}^j(M,A)=0$ for $j \neq 2$ if, and only if, M has Gelfand–Kirillov dimension 2. The modules that satisfy these equivalent conditions are called *holonomic*.

Since we are interested in non-holonomic modules, we must see what happens when $k=1$. Let a be a non-zero element of A and set $M = A/Aa$. Since M has a free resolution of length 1, it follows that $\mathrm{Ext}^j(M,A)=0$ if $j \neq 1$. Moreover, a simple calculation shows that M has dual $M^\star \cong A/A\tau(a)$. The main result of this section is that there exist irreducible modules of the form A/Aa whose dual is *not* irreducible.

Let $p \in \boldsymbol{C}^2$ and let d be a derivation of $\boldsymbol{C}[x_1,x_2]$. Denote by $\boldsymbol{m}$ the maximal ideal of $\boldsymbol{C}[x_1,x_2]$ corresponding to p under the Nullstellensatz. If p is a singularity of d then $\boldsymbol{m}$ is invariant under d. The 1-*jet* of d at p is the linear operator of $\boldsymbol{m}/\boldsymbol{m}^2$ induced by d. Let $d = g_1\partial_1 + g_2\partial_2$. Denote by J_d the transpose of the Jacobian matrix of the map of $\boldsymbol{C}^2$ to itself with coordinate functions (g_1, g_2). If $p = (\alpha_1, \alpha_2)$ then the images of $x_1 - \alpha_1$ and $x_2 - \alpha_2$ form a basis of $\boldsymbol{m}/\boldsymbol{m}^2$. The matrix of the 1-jet of d at p in this basis is equal to $J_d(p)$.

Suppose that the 1-jet of d at p has eigenvalues λ_1 and λ_2. We will denote by $\mathscr{L}(p)$ the lattice $\mathbf{Z}\lambda_1+\mathbf{Z}\lambda_2$. The *positive cone* $\mathscr{L}^+(p)$ of this lattice is the set $\{a\lambda_1+b\lambda_2\colon a,b \text{ are positive integers}\}$.

Lemma 2.1. *Let d be a derivation and let f be a polynomial in $\mathbf{C}[x_1,x_2]$. Let J be a left ideal of A. Assume that, for some point $p\in\mathbf{C}^2$,*
(1) *p is a singular point of d;*
(2) *the eigenvalues of the 1-jet of d at p are distinct;*
(3) *$d+f\in J$;*
(4) *A/J is an irreducible module supported at p.*
Then $f(p)\in\mathscr{L}^+(p)$.

Proof. Let λ_1 and λ_2 be the eigenvalues of $J_d(p)$. By translating p, we can assume that it is the origin. Moreover, since $\lambda_1\neq\lambda_2$, it follows that the matrix $J_d(p)$ is diagonalizable. Thus, a linear change of variables allows us to write d in the form

$$d=(g_1+\lambda_1x_1)\partial_1+(g_2+\lambda_2x_2)\partial_2,$$

where g_1 and g_2 have degree ≥ 2. Both the translation and the linear change of variables induce automorphisms in A which preserve $\mathbf{C}[x_1,x_2]$; see [5, Section 3, Ch. 1]. Note that hypotheses (1) to (4) are not affected by these automorphisms. Thus, without loss of generality, we can assume that p is the origin and that d has the form above.

Since A/J is irreducible and supported at the origin, it follows from *Kashiwara's equivalence* that A/J is isomorphic to $\mathbf{C}[\partial_1,\partial_2]$, the vector space generated by the monomials on ∂_1 and ∂_2 with the natural actions. For details see [5, Section 3, Ch. 18].

Let $H(m)$ be the vector subspace of $\mathbf{C}[\partial_1,\partial_2]$ generated by the monomials of degree $\leq m$. A calculation shows that if $r+s=m$, then

$$d(\partial_1^r\partial_2^s)\equiv -[\lambda_1(r+1)+\lambda_2(s+1)]\partial_1^r\partial_2^s \pmod{H(m-1)}.$$

In particular, $d\cdot H(m)\subseteq H(m)$. Let u be the image of $1+J$ in $\mathbf{C}[\partial_1,\partial_2]$ under the above isomorphism, and assume that u has degree m. Since $(d+f)(1+J)=0$ we conclude that

$$du\equiv -f(0)u \pmod{H(m-1)}. \tag{2.2}$$

But u has degree m as a polynomial in ∂_1 and ∂_2, thus

$$u\equiv\sum_{r+s=m}\alpha_{rs}\partial_1^r\partial_2^s \pmod{H(m-1)}.$$

Hence,

$$du\equiv -\sum_{r+s=m}\alpha_{rs}[\lambda_1(r+1)+\lambda_2(s+1)]\partial_1^r\partial_2^s \pmod{H(m-1)}.$$

The set $\{\partial_1^r\partial_2^s\colon r+s=m\}$ is a basis of $H(m)/H(m-1)$. Thus from (2.2), $f(0)=(r+1)\lambda_1+(s+1)\lambda_2$ whenever α_{rs} is non-zero. Since r,s are non-negative integers, $f(0)\in\mathscr{L}^+(0)$. □

This will be used to prove the following theorem, which is a slightly modified version of [1, Section 4, Proposition 6]. We assume that A is endowed with its *filtration by order*. Thus, the characteristic variety of an A-module refers to the variety calculated using this filtration. For more details see [5, Chs. 7–11] or [3, Ch. V].

Theorem 2.3. *Let d be a derivation, and f a polynomial, of $\mathbf{C}[x_1,x_2]$. Denote by $\{p_1,\dots,p_n\}$ the singular points of d in $\mathbf{C}^2$. Suppose that*
(1) $n \geq 2$;
(2) *the eigenvalues of the 1-jet of d at p_i are distinct for $1 \leq i \leq n$;*
(3) *d is not tangent to any algebraic curve of $\mathbf{C}^2$;*
(4) $f(p_1) \notin \mathscr{L}(p_1)$;
(5) *$f(p_i) \notin \mathscr{L}^+(p_i)$ for $2 \leq i \leq n$.*
Then $M = A/A(d+f)$ is irreducible. Moreover, if $0 \notin \mathscr{L}^+(p_r)$ for some $r \geq 2$ and $f(p_r) = 0$ then $M^\star$ is GK-*critical, but not irreducible.*

Proof. The proof is essentially the same as that of [1, Section 4, Proposition 6]. It is not necessary to repeat it in detail, but we will sketch it in order to point out where it must be modified, because the hypotheses on f above are weaker than the ones in [1, Section 4, Proposition 6]. The proof can be divided into three parts:

(1) It is shown that a proper homomorphic image H of M must be holonomic, no matter which f is chosen. This is where condition (3) is used. We conclude from this part that if H is irreducible then its characteristic variety is either the zero section of the cotangent bundle $\mathrm{T}^\star\mathbf{C}^2$ or a fibre of this bundle.

(2) In this part we must show that H cannot have the zero section as its characteristic variety. This is where condition (4) is used. Note that for the proof of [1, Section 4, Proposition 6] to work it is enough to assume that this condition holds for only one singular point of d.

(3) We are left with only one possibility for the characteristic variety of H, namely, a fibre of $\mathrm{T}^\star\mathbf{C}^2$ over a point q of $\mathbf{C}^2$. We must check that this cannot happen, since condition (5) is weaker than the hypothesis of [1, Section 4, Proposition 6]. Note that q must be stable under d; so it has to be a singular point of d. Hence, H is supported at one of the points $p_1,\dots,p_n$. But by Lemma 2.1 this is possible only if $f(p_i) \in \mathscr{L}^+(p_i)$, which is excluded by (4) and (5).

Thus, M has no non-trivial quotients, from which we deduce that it is irreducible. We still have to consider what happens to the dual of M, under the assumption that $0 \notin \mathscr{L}^+(p_r)$ and $f(p_r) = 0$ for some $r \geq 2$. As we saw above, the dual of M is $M^\star \cong A/A(d^\tau + f)$. Write $\boldsymbol{m}_r$ for the maximal ideal that corresponds to p_r. If $d = g_1\partial_1 + g_2\partial_2$ then we have that

$$d^\tau + f = -(\partial_1 \cdot g_1 + \partial_2 \cdot g_2) + f \in A\boldsymbol{m}_r$$

because $f(p_r) = 0$. Thus $M^\star$ has $A/A\boldsymbol{m}_r$ as a homomorphic image. In particular, M has a holonomic quotient module. On the other hand,

$$d^\tau + f = -(d + (\mathrm{tr}(J_d) - f)), \tag{2.4}$$

where $\mathrm{tr}(J_d)$ is the trace of the matrix J_d. Now applying the first part of the proof to $M^\star$, we conclude that its proper quotients must have Gelfand–Kirillov dimension 2. Since $M^\star$ itself has Gelfand–Kirillov dimension 3 we have proved that it is GK-critical. □

Let us now consider what happens to the dual of M if we assume that f satisfies the stronger hypothesis of [1, Section 4, Proposition 6]. We shall retain the notation of Theorem 2.3 for the rest of this section.

Scholium 2.5. *If $f(p_i) \notin \mathscr{L}(p_i)$ for $1 \le i \le n$ then $M^\star$ is irreducible.*

Proof. From (2.4)

$$M^\star \cong A/A(d + \mathrm{tr}(J_d) - f).$$

Since $\mathrm{tr}\, J_d(p_i) \in \mathscr{L}(p_i)$ but $f(p_i) \notin \mathscr{L}(p_i)$, it follows that $(\mathrm{tr}\, J_d - f)(p_i) \notin \mathscr{L}(p_i)$. Thus, we can apply Theorem 2.3 to $M^\star$ and deduce that it is irreducible. Note that we must assume that $f(p_i)$ does not belong to the whole lattice $\mathscr{L}(p_i)$, and not just the positive cone, because we go from f to $-f$ when we pass from M to its dual. □

Up till now, we have given no examples of derivations satisfying all these conditions. Let us now consider this question. The examples of [1, Section 4] are generic derivations; we will turn instead to singular foliations of projective space as a source of examples.

Let z_0, z_1, z_2 be homogeneous coordinates in the complex projective space $\boldsymbol{P}^2$. Identify the affine open set $z_0 \neq 0$ with $\boldsymbol{C}^2$ and put $x_i = z_i/z_0$ for $i = 1, 2$. A derivation $g_1\partial_1 + g_2\partial_2$ of $\boldsymbol{C}[x_1, x_2]$ can be associated to a 1-form $g_2 dx_1 - g_1 dx_2$ in a canonical way. We will assume that the polynomials g_1 and g_2 are co-prime and that $k = \max\{\deg g_1, \deg g_2\}$. The pull back of this form to $\boldsymbol{C}^3 \setminus \{0\}$ can be written in the form $\omega = \sum_0^2 A_i\, dz_i$, where the A_i are homogeneous polynomials of degree $k+1$ and $\sum_0^2 z_i A_i = 0$. Such a 1-form determines a singular foliation of $\boldsymbol{P}^2$.

Suppose that $p \in \boldsymbol{C}^2$ is a singular point of ω. Hence, p is a singular point of d. If the ratio of the eigenvalues of the 1-jet of d at p is *not* a positive real number then p is a singularity of *Poincaré type* of ω. A *Poincaré foliation* is one all of whose singular points are of Poincaré type. If ω is Poincaré then its *degree* is k.

Let V_k be the set formed by triples (A_1, A_2, A_3) of homogeneous polynomials of degree $k+1$ such that $\sum_0^2 z_i A_i$ is identically zero. Two such triples which differ by multiplication by a non-zero scalar define the same foliation in $\boldsymbol{P}^2$. This suggests that we look at the projective space $\boldsymbol{P}(V_k)$. Let $k \ge 2$ be a positive integer. It can be proved that the set $\mathscr{A}_k$ of Poincaré foliations of degree k in $\boldsymbol{P}^2$ that do not leave any algebraic curve invariant is an open and dense subset of $\boldsymbol{P}(V_k)$ in the analytic topology. See [8, Theorem B; 4, Section 4] for details.

Thus, if a derivation d of $\boldsymbol{C}[x_1, x_2]$ gives rise to a Poincaré foliation, it has no invariant algebraic curves. Two other conditions of Theorem 2.3 are automatically satisfied in this case. First, the eigenvalues of the 1-jet of d at any singular point must be non-zero and distinct, which follows from the definition of a Poincaré foliation. Moreover, if a foliation is Poincaré then it must have exactly k^2+k+1 singularities by [8, Section 3, Lemma 4]. Some of these can be at the line at infinity. But by Bézout's theorem, the number of points of intersection of the line at infinity with the curve $A_0=0$ cannot exceed $k+1$. Thus, d has at least k^2 singular points in $\mathbf{C}^2$. Since $k\geq 2$, it follows that d has at least 4 singular points.

An example of a foliation that satisfies all these requirements was given by Jouanolou in [7, p. 157]. This foliation is defined on the affine open set $z_0\neq 0$ by the derivation

$$\Delta=(1-x_1x_2^k)\partial_1+(x_1^k-x_2^{k+1})\partial_2. \tag{2.6}$$

An easy calculation shows that Δ has k^2+k+1 singular points in $\mathbf{C}^2$ and that the ratio of the eigenvalues of the 1-jets of d at each of these points is *not* a real number. For more details about this example, and for a proof that its foliation belongs to $\mathscr{A}_k$, see [8, Section 3.2; 4, Section 4; 7, p. 157 ff].

Note also that it is easy to construct a polynomial f that satisfies hypotheses (4) and (5) of Theorem 2.3. For $2\leq i\leq n$, let ϕ_i denote a linear polynomial such that $\phi_i(p_i)=0$ but $\phi_i(p_1)\neq 0$. Let α be a complex number linearly independent over $\mathbf{Q}$ with the eigenvalues of the 1-jet of d at p_1. Then

$$f=\frac{\alpha\phi_2\dots\phi_n}{\phi_2(p_1)\dots\phi_n(p_1)} \tag{2.7}$$

satisfies $f(p_1)=\alpha$ and $f(p_i)=0$, for $i\geq 2$. Using the Jouanolou example and this polynomial, we can give a more concrete version of Theorem 2.3.

Corollary 2.8. *Let $k\geq 2$ and $n=k^2+k+1$. Suppose that*

(1) *Δ is the derivation defined in* (2.6) *and $p_1,\dots,p_n$ are its singular points;*

(2) *f is the polynomial defined in* (2.7).

Then $M=A/A(\Delta+f)$ is irreducible. Moreover, $M^\star$ is GK-*critical, but not irreducible.*

Proof. This is a consequence of Theorem 2.3, and the properties of Δ and f described above. Note that $0\notin\mathscr{L}^+(p_i)$, for $1\leq i\leq n$, since the ratio of the eigenvalues of the 1-jet of Δ at each singular point is *not* a real number. □

3. Critical modules

We will now apply Theorem 2.3 to show that there exist critical modules of length 2 over A. First we need a lemma.

Lemma 3.1. *Let M be a finitely generated A-module such that* $\mathrm{Ext}^j(M,A)=0$ *if $j\neq 1$ and let N be a holonomic A-module. Then*

$$\mathrm{Ext}^1(N,M)\cong\mathrm{Hom}_A(M^\star,N^\star).$$

Proof. We will use the following result: suppose that U and V are left A-modules such that $\mathrm{Ext}^s(U,A)=0$, whenever $s\neq j$, then

$$\mathrm{Ext}^{j-t}(U,V)\cong \mathrm{Tor}_t(\mathrm{Ext}^j(U,A),V) \tag{3.2}$$

for all $0\leq t\leq j$. This is proved in [2, Ch. 2, Proposition 4.12]. Since N is holonomic, it follows from (3.2) that

$$\mathrm{Ext}^1(N,M)\cong \mathrm{Tor}_1(\mathrm{Ext}^2(N,A),M).$$

But

$$\mathrm{Tor}_1(\mathrm{Ext}^2(N,A),M)\cong \mathrm{Tor}_1(M^\tau,N^\star).$$

Since M satisfies $\mathrm{Ext}^j(M,A)=0$ for $j\neq 1$ and $M^\tau\cong \mathrm{Ext}^1(M^\star,A)$, the desired result is obtained from another application of (3.2). □

We can now state the main result of this paper. As in Section 2 we will denote the maximal ideal of $\boldsymbol{C}[x_1,x_2]$ that corresponds to the point p_i of $\boldsymbol{C}^2$ by $\boldsymbol{m}_i$.

Theorem 3.3. *Let d be a derivation and f a polynomial of $\boldsymbol{C}[x_1,x_2]$. Denote by $\{p_1,\dots,p_n\}$ the singular points of d in $\boldsymbol{C}^2$. Suppose that*

(1) $n\geq 2$;

(2) *the eigenvalues of the* 1*-jet of d at p_i are linearly independent over $\boldsymbol{Q}$ for* $1\leq i\leq n$;

(3) *d is not tangent to any algebraic curve of $\boldsymbol{C}^2$*;

(4) $f(p_1)\notin \mathscr{S}(p_1)$;

(5) $f(p_r)=0$ *for some* $2\leq r\leq n$.

Then $\mathrm{Ext}^1(A/A(d+f),A/A\boldsymbol{m}_r)\neq 0$.

Proof. Note that in (2) we now require the eigenvalues to be linearly independent over $\boldsymbol{Q}$. This guarantees that $0\notin \mathscr{S}^+(p_r)$. Together with (5), this allows us to conclude from Theorem 2.3 that $A/A\boldsymbol{m}_r$ is a quotient of $M^\star$. Note that the dual of $A/A\boldsymbol{m}_r$ is irreducible and supported at p_r. Thus, $A/A\boldsymbol{m}_r$ is isomorphic to its dual. The result now follows by Lemma 3.1. □

The Jouanolou example Δ of Section 2 satisfies all the hypothesis above. Actually the hypothesis (2) holds generically in the space of foliations $\boldsymbol{P}(V_k)$. Since the Poincaré foliations that have no invariant curve form an open and dense subset of this space, we are assured of an abundance of derivations satisfying all the hypothesis of Theorem 3.3. Moreover, as we have also seen in Section 2, it is easy to construct a polynomial f that satisfies hypotheses (4) and (5). Thus critical A-modules of length 2 do exist. Building on this approach we can actually construct an explicit example.

Corollary 3.4. *Let d and f be as in Theorem 3.3, and put $M = A/A(d+f)$. Let N be a submodule of $M^\star$ such that $M^\star/N$ is irreducible. Then $N^\star$ is a critical module of length 2 over A.*

Proof. Write $H = M^\star/N$. Since $M^\star$ is GK-critical, H is holonomic and the sequence

$$0 \to N \to M^\star \to H \to 0 \tag{3.5}$$

is non-split exact. Moreover, N must be GK-critical of dimension 3. From the explicit characterization of $M^\star$ obtained in Section 2, we conclude that $\mathrm{Ext}^j(M^\star, A) = 0$ if $j \neq 1$. Since H is holonomic, it satisfies $\mathrm{Ext}^j(H, A) = 0$ whenever $j \neq 2$. Using these two facts and the long exact sequence of Ext groups, we obtain the following exact sequence of right A-modules

$$0 \to \mathrm{Ext}^1(M^\star, A) \to \mathrm{Ext}^1(N, A) \to \mathrm{Ext}^2(H, A) \to 0,$$

and also the fact that $\mathrm{Ext}^j(N, A) = 0$ for $j \neq 1$.

From the explicit description of M it is easy to see that $(M^\star)^\star \cong M$. Thus, by transposing the actions, one obtains an exact sequence of left A-modules

$$0 \to M \to N^\star \to H^\star \to 0, \tag{3.6}$$

where $H^\star$ denotes the dual in the holonomic category. Since H is irreducible, so is $H^\star$. But M is irreducible, hence $N^\star$ has length 2.

If $j \neq 1$ then $\mathrm{Ext}^j(N, A) = 0$, and so $\mathrm{Ext}^j(N^\star, A) = 0$. This implies that (3.6) does not split. Indeed, from $N^\star \cong H^\star \oplus M$ it follows that

$$0 = \mathrm{Ext}^2(N^\star, A)^\tau \cong H,$$

which is a contradiction. Thus (3.6) is non-split. But this implies that if S is a non-zero submodule of $N^\star$ then $S \cap M \neq 0$. Since M is irreducible we have $M \subseteq S$. In particular, $N^\star$ is a critical module, as required. □

Note that if the singular point p_r of Theorem 3.3 has coordinates (α_1, α_2), then

$$N = \frac{A(x_1 - \alpha_1) + A(x_2 - \alpha_2)}{A(d + \mathrm{tr}\, J_d - f)}.$$

This gives a fairly explicit description of the critical module of length 2.

Finally, note that it is not clear whether these results can be extended to the nth Weyl algebra A_n, when $n \geq 3$. The fact that we are working over the second Weyl algebra is used several times in the paper; especially in Lemma 3.1 and the first part of the proof of Theorem 2.3. It is not clear how to get around these difficulties.

Acknowledgements

I wish to thank J.T. Stafford for drawing my attention to Perets' paper and also W. Zuñiga, M.P. Holland and M.I. Camacho for many helpful suggestions and comments. The work on this paper was partially supported by a grant from the Brazilian National Research Council (CNPq).

References

[1] J. Bernstein, V. Lunts, On non-holonomic irreducible D-modules, Invent. Math. 94 (1988) 223–243.
[2] J.-E. Björk, Rings of Differential Operators, North-Holland, Amsterdam, 1979.
[3] A. Borel et al., Algebraic D-modules, in: Perspectives in Mathematics, vol. 2, Academic Press, Orlando, 1987.
[4] D. Cerveau, A. Lins Neto, Holomorphic foliations in $CP(2)$ having an invariant algebraic curve, Ann. Inst. Fourier, Grenoble 41 (1991) 883–903.
[5] S.C. Coutinho, A primer of algebraic D-modules, in: London Mathematical Society Student Texts, vol. 33, Cambridge University Press, Cambridge, 1995.
[6] R. Gordon, J.C. Robson, Krull Dimension. Mem. Amer. Math. Soc. 133, American Mathematical Society, Providence, RI, 1973.
[7] J.C. Jouanolou, Equations de Pfaff algébriques, in: Lecture Notes in Mathematics, vol. 708, Springer, Berlin, 1979.
[8] A. Lins Neto, Algebraic solutions of polynomial differential equations and foliations in dimension two, in: Holomorphic Dynamics, Lecture Notes in Mathematics, vol. 1345, Springer, 1988, Berlin, pp. 192–232.
[9] G.S. Perets, d-Critical modules of length 2 over Weyl algebras, Israel J. Math. 83 (1993) 361–368.
[10] J.T. Stafford, Non-holonomic modules over Weyl algebras and enveloping algebras, Invent. Math. 79 (1985) 619–638.